BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY.

CONTINUATION OF THE BULLETIN OF THE NEW YORK MATHEMATICAL SOCIETY.

A HISTORICAL AND CRITICAL REVIEW OF MATHEMATICAL SCIENCE.

EDITED BY

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BULLETIN OF THE

AMERICAN MATHEMATICAL SOCIETY

THE SUMMER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE first summer meeting of the AMERICAN MATHEMATICAL SOCIETY was held in one of the lecture-rooms of the Polytechnic Institute in Brooklyn, N. Y., on Tuesday and Wednesday, August 14 and 15, 1894. These days immediately preceded those fixed for the meeting of the American Association for the Advancement of Science, and all concessions obtained by that Association from railways and hotels were extended to the members of the Mathematical Society. The members of the Society were invited to participate also in the entertainments and excursions arranged in honor of the Association. To the co-operation of that larger organization thus was due in part the success of the Society's meeting. Among those present were Dr. E. M. Blake, Professor Henry B. Fine, Professor Thomas S. Fiske, Professor G. B. Halsted, Dr. George W. Hill, Professor E. W. Hyde, Mr. P. A. Lambert, Dr. Emory McClintock, Professor Alexander Macfarlane, Professor Mansfield Merriman, Professor E. Hastings Moore, Professor Frank Morley, Professor Hubert A. Newton, Professor George D. Olds, Mr. J. C. Pfister, Mr. Ralph A. Roberts, Professor James B. Shaw, Jr., Mr. J. F. Shields, Professor Ormond Stone, Dr. Edward B. Van Vleck, Professor L. G. Weld, Professor R. S. Woodward, and Professor Alexander Ziwet. Although the attendance was not so large as had been anticipated, the interest was well maintained throughout. On Wednesday evening, after adjournment, a dinner was given at the St. George Hotel, in which a majority of the members who had attended the meeting participated.

The Society elected the following new members: Rev. John G. Hagen, S.J., Georgetown College Observatory, West Washington, D.C.; Dr. Gustavus Hay, Boston, Mass.; Professor Hubert A. Newton, Yale University, New Haven, Conn.; Professor Harry Walter Tyler, Massachusetts Institute of Technology, Boston, Mass.

The following papers were presented:

1. Theorems in the calculus of enlargement. Dr. Emory McClintock, New York, N. Y.

2. A method for calculating simultaneously all the roots of an equation. Dr. Emory McClintock, New York, N. Y.

3. Elliptic functions and the Cartesian curve. Professor Frank Morley, Haverford, Pa.

4. Concerning the definition by a system of functional properties of the function $f(z) = \frac{\sin \pi z}{\pi}$ Professor E. Hastings Moore, Chicago, Ill.

5. Bertrand's paradox and the non-euclidean geometry. Professor George Bruce Halsted, Austin, Texas.

6. Analytical theory of the errors of interpolated values from numerical tables. Professor R. S. Woodward, New York, N. Y.

7. Upon the problem of the minimum sum of the distances of a point from given points. Professor V. Schlegel, Hagen, Germany.

8. On the fundamental laws of algebra. Professor Alexander Macfarlane, Austin, Texas.

9. About cube numbers whose sum is a cube number. Dr. Artemas Martin, Washington, D.C.

10. Reduction of the resultant of a binary quadric and n-ic by virtue of its semicombinant property. Professor Henry S. White, Evanston, Ill.

In the absence of their authors, paper No. 7 was presented by Professor Hyde, paper No. 9 by the Secretary, and No. 10 by Professor Ziwet.

Nos. 1, 2, 3, are intended for publication in the *American Journal of Mathematics*; Nos. 4, 6, in the *Annals of Mathematics*; No. 5 in the *Proceedings of the Texas Academy of Science*; Nos. 7, 10, in the *Bulletin of the American Mathematical Society*; No. 9, in the *Mathematical Magazine*.

In the first of Dr. McClintock's papers the formulæ developed are all special cases of the most general theorem presented, viz.:

$$fE = fz + fE\,0^{[1]} \cdot H + fE_0 0^{[2]} \cdot H^2/2! + \ldots.$$

where E is an operation such that, applied upon any function, say ψx, we shall have $E^h \psi x = \psi(x + h)$; H is a function of E depending on some other function ϕE and defined by $H h \phi E = E^h - z^h$; $fE_0 0^{[m]}$ means $fE_x x^{[m]}{}_{[x=0]}$; and $x^{[m]} = x(\phi E_x)^m z^{x-mh}(x-h)(x-2h)\ldots(x-mh+h)$. Since $\psi E_0 y^0 = \psi y$, an important case, derived by operating with the chief series upon y^0, is

$$fy = fz + fE_0 0^{[1]} \cdot x + fE_0 0^{[2]} \cdot x^2/2! + \ldots,$$

where x is the same function of y as H of E, defined by $xh\phi y = y^h - z^h$. If, for a special case of this, $h = 1$, the result is identical in value with Lagrange's theorem, but with no reference to differentiation in the form of the coefficients.

The second paper contains a remarkable application of the series last quoted to the theory of equations. The general formula, applicable to any equation $x^n = c + n\phi x$, is, after putting $\omega^n = c$,

$$x = \omega + \omega^{1-n}\phi\omega + E_\omega(\phi E_\omega)^2\omega^{2-2n}(0-n)/2! + E_\omega(\phi E_\omega)^3\omega^{3-3n}(0-n)(0-2n)/3! + \ldots$$

The simplest form of c is 1 or -1, and when the equation is arranged in that form this series, if convergent, gives all the roots upon substituting for ω the several nth roots of 1 or of -1, as the case may be. If c is not 1 or -1, it will be $c = \omega^n = \zeta^n\xi^n$, where ζ^n is the numerical value of c, and ξ^n is 1 or -1, and in this case ω is to be replaced by $\zeta\xi$, ξ having the several n values of the roots of 1 or of -1. The formula holds when n has any value, positive or negative, though nothing is gained by adjusting the equation with n negative or fractional; and in particular the term chosen for x^n need not be the highest power present, though in that case, of course, only n roots will be developed by the series at once. Especial attention is given to the trinomial equation $x^n = \omega^n + nax^{n-k}$, wherein k is not necessarily positive. For this equation the general formula becomes

$$x = \omega + \omega^{1-k}\,.\,a + \omega^{1-2k}(1-2k+n)\,.\,a^2/2! + \omega^{1-3k}(1-3k+n)(1-3k+2n)\,.\,a^3/3! + \ldots$$

This formula, though actually first obtained as here described, may be derived from Lagrange's theorem, by putting $x^n = u$, $\omega^n = z$, $\phi x = \phi(u^{\frac{1}{n}})$, and expanding $u^{\frac{1}{n}}$. Similarly, the general formula may be used in this alternative form, likewise derived, as an afterthought, from Lagrange's theorem:

$$x = \omega + \omega^{1-n}\phi\omega + \omega^{1-n}\frac{d}{d\omega}\omega^{1-n}(\phi\omega)^2/2! + (\omega^{1-n}\frac{d}{d\omega})^2\omega^{1-n}(\phi\omega)^3/3! + \ldots$$

The trinomial formula is convergent whenever n is positive and the numerical value of a^n is less than that of

$$k^{-k}(n-k)^{k-n}\omega^{nk}.$$

Its use is illustrated by the complete exhibition of the six imaginary roots of $x^6 + x + 1 = 0$. When n is negative, the trinomial formula gives the same series as if both sides of the equation were multiplied by x^{-n}, $x^n = \pm 1 + nax^{n-k}$

becoming $x^{-n} = \pm 1 \mp nax^{-k}$. If the value of a is such that the series for the trinomial $x^n = \pm 1 + nax^{n-k}$ is not convergent, it is shown that the series for $x^{n-k} = (\mp 1 + x^n)/na$ must be convergent, yielding $n - k$ of the n roots, and that the other k roots may be obtained either by putting $x = u^{-1}$ and multiplying throughout by u^n, whence $u^k = (1 \mp u^n)/na$, or by using $x^k = na \pm x^{k-n}$, both forms yielding convergent series. When the reciprocal transformation $x = u^{-1}$ is employed, it is convenient to use one of two formulæ;* viz., for the trinomial $u^n = \omega^n + nau^{n-k}$,

$$x = u^{-1} = \omega^{-1} - \omega^{-1-k} . a - \omega^{-1-2k}(-1-2k+n) . a^2/2! - \omega^{-1-3k}(-1-3k+n)(-1-3k+2n) . a^3/3! - \ldots,$$

and for the general equation $u^n = \omega^n + n\phi u$,

$$x = u^{-1} = \omega^{-1} - \omega^{-1-n}\phi\omega - E_\omega^{-1}(\phi E_\omega)^2\omega^{p-2n}(0-n)/2! - \ldots$$

Professor Morley's paper aims at showing how the properties of the so-called Cartesian oval, both those that are known and others, can be systematically and easily developed by means of Weierstrass's elliptic functions. The following topics are considered : The Cartesian oval as a normal elliptic curve. The relations between distances of a point from the single foci, the triple focus, and the double tangent. Abel's theorem for elliptic functions of the second kind. The intersection of a circle and the curve. The bitangent circles. The circles of curvature. The tangents from a point.

The system of functional properties receiving principal consideration in Professor Moore's paper is the following : (1) $f(z)$ is an integral analytic function of the complex variable z. It has as its complete system of zeros, $z = m$, $m = 0, \pm 1, \pm 2, \pm 3, \ldots$, the multiplicity of each zero being unity, and limit when $z = 0$ of $f(z)/z = 1$. (2) $f(2z)f(\frac{1}{2}) = 2f(z)f(z + \frac{1}{2})$. (3) $f(-z) = -f(z)$. This furnishes a new determination of the external exponential factor ($e^{\pi z i} = 1$) in the expression of the function $f(z)$ as a Weierstrassian infinite product :

$$f(z) = \frac{\sin \pi z}{\pi} = e^{\varphi(z)} z \prod_m{}' \left(1 - \frac{z}{m}\right) e^{\frac{z}{m}} \qquad [m \neq 0]$$

* These reciprocal formulæ have been communicated to the Editors of the Bulletin by Dr. McClintock, having been devised by him shortly after reading the paper before the Society. He has undertaken to read a second paper at the next meeting, in which he will indicate the manner whereby, by having regard to the larger or "dominant" coefficients, any equation may apparently be so adjusted as to yield one or more convergent series for all the roots, and will illustrate the separation of roots by this method by all of the many examples employed for the application of Sturm's theorem in Burnside & Panton's Theory of Equations. Equations having imaginary coefficients will also be illustrated, as to which at the reading, in reply to a question, it was said that such equations, though not yet examined, must certainly come within the theory.

Professor Halsted's paper gives, after a historical introduction, Bertrand's renowned paradox in a new form, and maintains that it is inexplicable without recourse to the non-euclidean geometry, but with the aid of Lobachevsky's theory of the equidistantial is readily shown to involve a *petitio principii*.

The object of Professor Woodward's paper is to extend and complete the theory of the errors in question. The investigation is supplementary to papers already published by the author on this subject in the *Analyst*, 1882, and in the *Annals of Mathematics*, 1886. The special peculiarity of the investigation is that it requires the combination of continuous and discontinuous errors. The quasi-geometrical method followed in the previous papers is now supplanted by a wholly analytical process.

In Professor Schlegel's paper the differential equation between the distances $p_1, p_2, \ldots p_n$ of a variable point P from n fixed points in an $(n-1)$-*dimensional space* is first derived and the minimum of the function

$$s = \lambda_1 p_1^{\alpha} + \lambda_2 p_2^{\alpha} + \ldots + \lambda_n p_n^{\alpha}$$

of these distances is determined. The general result thus obtained (I) reduces in the case of the plane to an analytical form given by Steiner and furnishes in the case of ordinary space the geometrical condition found by Sturm. To extend the problem to the case of n points in a *space of less than* $n-1$ *dimensions* and to gain at the same time a purely geometrical interpretation of the general condition for a minimum, the formula (I) is put into another form (II); then the generalized problem is solved by a formula (III) derived by a different method; this formula (III) is interpreted geometrically and finally shown to reduce to formula (II). Thus the organic connection is established between the general methods of which the results found by Steiner, Sturm, and others are particular cases. Some special cases are next discussed and remarks are made concerning the construction of the minimum point and the formation of the point-system belonging to a given minimum point. The case of a linear row of points is specially investigated and applied to the solution of a practical problem. Finally the results are applied to the problem in mechanics of determining the point of least attraction for a given group of points. The investigation is throughout carried on by the methods of *Grassmann's geometrical calculus*.

Professor Macfarlane maintains that the laws of algebra ought to be placed on a real basis, not on a formal basis. The several formal laws are examined, and it is considered whether they are entirely independent. Their validity in space analysis is considered, the exact conditions where some of them break down being pointed out.

Dr. Martin deduces in his paper a number of formulæ which lead to very remarkable numerical results.

In Professor White's paper the problem is to reduce the resultant of a binary quadric and n-ic to a sum of products of invariants of the "reduced system." Clebsch, and later Gordan, have solved it. They use unnecessary auxiliaries. The reduction is accomplished more speedily by constructing synthetically, according to the method of the theory of forms, an expression involving undetermined coefficients whose values are then found by the differential equation of apolarity applied to that covariant which becomes an exact power of the common factor of the two quantities in case their resultant vanishes. Incidentally the connection between apolarity and the semicombinant property is illustrated.

THOMAS S. FISKE.

ON THE CONNECTION BETWEEN BINARY QUARTICS AND ELLIPTIC FUNCTIONS.*

BY PROFESSOR E. STUDY.

AFTER the treatment which the subject has received in the recent work of Professors Harkness and Morley, it may be supposed that American mathematicians are quite familiar with the utilization of the theory of invariants in the theory of elliptic functions. We shall confine ourselves to the simplest case, where the theory of a binary quartic is concerned. We intend to show how a certain group of rational and irrational covariants of a binary quartic can be expressed as one-valued functions of one or two parameters, thus filling up a number of lacunae contained in former presentations of the subject.

After having explained the system of notation we are to apply, we proceed, partly following Cayley, to define a system of irrational covariants. These being known, we compare these quantics with the elliptic θ-functions; and thus we will be enabled to express the connection in question in very simple terms.

Of course we must suppress here not only the proofs, but also quite a number of details; a full exposition will be published shortly in the *American Journal of Mathematics*.

1. *Notation.*

Denoting the quartic $f(x)$ symbolically by $f = (ax)^4 = (a_1x_2 - a_2x_1)^4$, the following forms constitute, as is well known, what is termed the complete system of f:

* This paper, which Professor Study kindly transmitted for presentation at the Brooklyn meeting of the AMERICAN MATHEMATICAL SOCIETY, arrived too late for that purpose.

$$\left.\begin{array}{ll} f &= (ax)^4 \\ h &= \frac{1}{4}(ab)^2(ax)^2(bx)^2 = (hx)^4 \\ t &= 2(ah)(ax)^2(hx)^3 = (tx)^6 \\ g_2 &= \frac{1}{4}(ab)^4 \\ g_3 &= \frac{1}{8}(ah)^4 = \frac{1}{4}(ab)^2(ac)^2(bc)^2 \end{array}\right| \left.\begin{array}{l} f \\ \frac{H}{2} \\ T \\ \frac{i}{2} \\ \frac{j}{6} \end{array}\right\} \text{(Notation applied by Clebsch.)}$$

$$G = \tfrac{1}{128}(g_2^3 - 27g_3^2)$$

is the discriminant.

2. *Decomposition of f into its linear factors.*

In order to decompose f into linear factors, which are to be *irrational covariants* of f, we have first to decompose the Jacobian t into its three conjugate quadratic factors by means of the solution of the cubic equation

$$4x^3 - g_2x - g_3 = 0.$$

We denote the roots by e_λ, e_μ, e_ν. $h + e_\lambda f$ being a perfect square, we are enabled to define a set of three quadratic forms l, m, n by means of the formulæ

$$l = (lx)^2 = \sqrt{\frac{h + e_\lambda f}{(e_\mu - e_\lambda)(e_\nu - e_\lambda)}}, \text{ etc.},$$

restricting the meaning of the radicals by the further supposition

$$(mn)(mx)(nx) = -(lx)^3, \text{ etc.}$$

Then we have

$$t = -2\sqrt{G} \,.\, (lx)^3 \,.\, (mx)^3 \,.\, (nx)^3.$$

Writing now

$$f = 4(r_\circ x) \,.\, (r_\lambda x) \,.\, (r_\mu x) \,.\, (r_\nu x),$$

we may define the linear forms $(r_\lambda x)$, irrational covariants of f, by means of the formulæ

$$\left\{\begin{array}{l} 2(r_\circ x)^2 = -\sqrt{e_\mu - e_\nu} \,.\, l - \sqrt{e_\nu - e_\lambda} \,.\, m - \sqrt{e_\lambda - e_\mu} \,.\, n, \\ 2(r_\lambda x)^2 = -\sqrt{e_\mu - e_\nu} \,.\, l + \sqrt{e_\nu - e_\lambda} \,.\, m + \sqrt{e_\lambda - e_\mu} \,.\, n, \\ 2(r_\mu x)^2 = \sqrt{e_\mu - e_\nu} \,.\, l - \sqrt{e_\nu - e_\lambda} \,.\, m + \sqrt{e_\lambda - e_\mu} \,.\, n, \\ 2(r_\nu x)^2 = \sqrt{e_\mu - e_\nu} \,.\, l + \sqrt{e_\nu - e_\lambda} \,.\, m - \sqrt{e_\lambda - e_\mu} \,.\, n, \end{array}\right.$$

the expressions on the right being again perfect squares.

By these formulæ, which are due to Cayley, the forms $(r_\lambda x)$ are not yet fully determined. In order to restrict their ambiguity as much as possible, we have to add some further conditions. These are furnished by a remarkable fact, which seems to have escaped notice hitherto: *The bilinear invariants* $(r_\iota r_\varkappa)$ *belong to the domain of rationality defined by the quantities* $\sqrt{e_\mu - e_\nu}$, *etc.*

As a close examination shows, it is possible to restrict the values of the quantities $(r_\varkappa x)$ by the further suppositions

$$\left\{\begin{array}{l} (r_\mu r_\nu) = -(r_\varrho r_\lambda) = \sqrt{e_\mu - e_\nu}, \\ (r_\nu r_\lambda) = -(r_\varrho r_\mu) = \sqrt{e_\nu - e_\lambda}, \\ (r_\lambda r_\mu) = -(r_\varrho r_\nu) = \sqrt{e_\lambda - e_\mu}. \end{array}\right.$$

The signs of the square roots on the right being chosen, and l, m, n being known, the linear factors $(r_\varkappa x)$ of f are completely determined, excepting a change of sign of the whole set, which remains still possible.

3. *Introduction of the θ-functions.*

Now an examination of the relations among the linear forms $(r_\varkappa x)$ furnishes the very important result, *that these relations are exactly identical with the relations among the squares of the four elliptic θ-functions*—provided that the definition of the θ-functions which has been given by the author in his paper (*American Journal*, vol. 16, No. 2) is accepted.

Identifying, therefore, the quantities $\sqrt{e_\mu - e_\nu}$, taken from the theory of the quartic, with the quantities $\sqrt{e_\mu - e_\nu} = \theta_\lambda(0)$, occurring in the theory of elliptic functions, we are enabled to explain a set of square roots by means of the formulæ

$$(1) \qquad \sqrt{(r_\varkappa x)} = \theta_\varkappa u \quad (\varkappa = o, \lambda, \mu, \nu),$$

$$(2) \qquad \sqrt{f} = -2\theta_o u \theta_\lambda u \theta_\mu u \theta_\nu u.$$

These formulæ contain the clue to the whole theory in question. It is easy now to derive all known results and a large number of new ones; and this is done by means of actual calculations, without any help of the so-called method of indeterminate coefficients.

We limit ourselves to communicating some of the most interesting results.

Passing from the single to the double argument, we obtain first

$$(3)\qquad \begin{cases} \sqrt{f} = -\theta' . \theta(2u), \\ l = -\theta_\lambda . \theta_\lambda(2u), \\ m = -\theta_\mu . \theta_\mu(2u), \\ n = -\theta_\nu . \theta_\nu(2u), \end{cases}$$

where θ_λ and θ' mean $\theta_\lambda(0)$ and $\theta'(0) = \theta_\lambda\theta_\mu\theta_\nu$; and considering two pairs of variables x, u and y, v at the same time, we have

$$(4)\qquad \begin{cases} (xy) = -\theta(u+v)\,\theta(u-v), \\ (lx)(ly) = -\theta_\lambda(u+v)\,\theta_\lambda(u-v), \\ (mx)(my) = -\theta_\mu(u+v)\,\theta_\mu(u-v), \\ (nx)(ny) = -\theta_\nu(u+v)\,\theta_\nu(u-v), \end{cases}$$

or expressions of the identical covariant $(xy) = x_1y_2 - x_2y_1$ and of the polars of the quadratic forms $(lx)^2$, $(mx)^2$, $(nx)^2$ in terms of u, v.

These formulæ, especially the equations (4), are not only worth noticing for their symmetry and elegance, but they are also very useful; they are the true central point of our theory. By the formulæ (4) the general addition theorems of Jacobi and Weierstrass are brought into close connection with the properties of the four bilinear forms on the left. It is remarkable that these very forms bear a simple relation still to another theory: the composition of our forms is exactly identical with the multiplication of the four *quaternion-units*.

4. *The elliptic functions* $\wp u$, $\wp' u$.

By means of the preceding formulæ Weierstrass' functions $\wp u$, $\wp' u$ are easily expressed in terms of covariants of f.

Supposing $\omega_0 = 0$, and denoting the periods corresponding to θ_λ, θ_μ, θ_ν, as usual by $2\omega_\lambda$, $2\omega_\mu$, $2\omega_\nu$, we have first a group of irrational formulæ:

$$(5)\qquad \begin{cases} \wp(u+\omega_\kappa) = \dfrac{(tr_\kappa)^4(tx)}{2\sqrt[4]{G}\,.\,(ar_\kappa)^3(ax)} = \dfrac{(tr_\kappa)^4(tx)}{2\sqrt{G}\,.\,(r_\kappa x)}, \\ \wp'(u+\omega_\kappa) = \sqrt[4]{G}\,.\,\dfrac{\sqrt{f}}{(r_0x)^3}, \\ \wp''(u+\omega_\kappa) = \dfrac{3(tr_\kappa)^4(tx)^2}{\sqrt[4]{G}\,.\,(r_\kappa x)^3}, \quad \text{etc.} \quad (\kappa = 0, \lambda, \mu, \nu.) \end{cases}$$

For the elliptic functions of the double argument, we tain simpler expressions, namely, Hermite's famous form

$$(6)\qquad \begin{cases} \wp(2u) = -\dfrac{h}{f}, \\ \wp'(2u) = \dfrac{t}{f\sqrt{f}}, \text{ etc.} \end{cases}$$

Considering again two pairs of variables, we find further remarkable formulæ

$$(7)\qquad \begin{cases} \wp(u \pm v) = \dfrac{(ax)^2(ay)^2 \mp \sqrt{(ax)^4}\,\sqrt{(ay)^4}}{2(xy)^2}, \\ \wp'(u \pm v) = -\dfrac{(ay)^2(ax).\sqrt{(ax)^4} \mp (ax)^2(ay)\sqrt{(ay)^4}}{(xy)^3}, \end{cases}$$

the content of which is partly due to Weierstrass.

It follows now that all covariants of the quartic f contain the two sets of variables (x_1, x_2) *and* (y_1, y_2) *in the degree z are rational functions of* $\wp(u+v)$, $\wp(u-v)$, $\wp'(u+v)$, $\wp'(u-v)$, *and vice versa*—provided we include in the variants of f the two irrationalities $\sqrt{f_x}$ and $\sqrt{f_y}$.

In particular, when a power of (xy) *is the sole denominat the covariant in question is an integral function of the fo elliptic functions mentioned.*

A few simple examples will show the significance of t last theorem. We have

$$\wp(u+v).\wp(u-v) + \tfrac{1}{12}g_2 = -\frac{(hx)^2(hy)^2}{(xy)^4},$$

$$\wp'(u+v).\wp'(u-v) = -2\frac{(tx)^3(ty)^3}{(xy)^6},$$

$$\wp(u \pm v).\wp'(u \mp v) = \frac{(hy)^2(hx).\sqrt{(ax)^4} \pm (hx)^2(hy)\sqrt{(ay)^4}}{(xy)^5}$$

$$\tfrac{1}{2}\wp'(u \pm v).\wp''(u \pm v) = \frac{(ty)^4(tx)^2.\sqrt{(ax)^4} \pm (tx)^4(ty)^2.\sqrt{(ay}}{(xy)^8}$$

etc

REDUCTION OF THE RESULTANT OF A BINARY QUADRIC AND n-IC BY VIRTUE OF ITS SEMI-COMBINANT PROPERTY.*

BY PROFESSOR HENRY S. WHITE.

THE resultant of two binary quantics is comparatively simple when written either in the Eulerian or in the Bezoutian form as a determinant. Its degree in the coefficients of each quantic, however, is generally high.

Hence the problem arises, when invariants are arranged according to their degrees in the coefficients: *To reduce the resultant to a sum of terms, each of which shall contain the simplest possible invariant factors.* This problem is completely solved in papers by Clebsch, Gordan, and E. Pascal. But much may yet be done in the way of simplifying the processes by which their results are deduced. I propose here to discuss the partial problem which Clebsch solved; namely, *to write in symbolic form the resultant of a binary quadric and a binary quantic of arbitrary order n.*

By a method different from his I shorten the labor involved and at the same time illustrate the utility of the theory of *conjugate forms.*

Let the quantics be denoted by

$$\phi = a_x^2 = a_1^2 x_1^2 + 2a_1 a_2 x_1 x_2 + a_2^2 x_2^2,$$

$$f = a_x^n = a_1^n x_1^n + n \cdot a_1^{n-1} a_2 x_1^{n-1} x_2 + \dots,$$

according to the Clebsch-Aronhold symbolism.

Then the resultant is written in Sylvester's formula:

$$R = \begin{vmatrix} a_1^n & na_1^{n-1}a_2 & \frac{n(n-1)}{1 \cdot 2} a_1^{n-2} a_2^2 \dots & a_2^n & 0 \\ 0 & a_1^n & na_1^{n-1}a_2 \dots & na_1 a_2^{n-1} & a_2^n \\ \alpha_1^2 & 2\alpha_1\alpha_2 & \alpha_2^2 \dots & 0 & 0 \\ 0 & \beta_1^2 & 2\beta_1\beta_2 \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 \dots & 2\epsilon_1\epsilon_2 & \epsilon_2^2 \end{vmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}a\\a\end{matrix}}\right\} 2 \text{ rows.} \\ \left.\vphantom{\begin{matrix}a\\a\\a\\a\\a\\a\end{matrix}}\right\} n \text{ rows.} \end{matrix}$$

* Presented to the AMERICAN MATHEMATICAL SOCIETY at its Summer Meeting, August 15, 1894.

This determinant exhibits plainly the "semicombinant" property of the resultant, viz., that it remains unaltered when the coefficients of f are replaced by those of F:

$$F = f + L\phi = A_x^n = a_x^n + \lambda_x^{n-2} \cdot \alpha_x^2$$

(in which formula the coefficients of λ_x^{n-2} are $n-1$ arbitrary quantities). For this substitution of A_x^n for a_x^n merely increases the coefficients standing in the first and second rows, respectively, of the determinant R by arbitrary multiples of those in the $(n-1)$ rows beginning with the third and the fourth rows respectively.

This derivation of the semicombinant property is so well known as to seem trivial. I adduce it here for the sake of showing, as a consequence of it, two properties of a certain covariant, derived by the evectant process from the resultant.

The evectant that I mean is that obtained by substituting, in every term of the developed determinant R for *one* of the two factors $a_1^i a_2^{n-i}$, or $a_1^k a_2^{n-k}$ the variable factor $x_2^i(-x_1)^{n-i}$, or $x_2^k(-x_1)^{n-k}$. If I make the substitution without expanding the determinant, it is done by substituting for the first row only, then for the second row only, in succession, the proper power-products of x_2 and $(-x_1)$, and adding the resulting terms. The covariant, of order n, so derived I will call $\overline{F}$, or $\overline{A}_x^n$ or $\overline{B}_x^n$. Now each of the two determinants composing $\overline{F}$ has obviously, like the resultant, the semicombinant property; therefore

(1) the covariant $\overline{F}$ is a semicombinant of f and ϕ.

But, further, if I replace in $\overline{F}$ the power-products of x_1, x_2 by their several coefficients in the symbolic product

$$(x_1 y_1 + x_2 y_2)^{n-2}(x_1\alpha_2 - x_2\alpha_1)^2,$$

linear and homogeneous in the coefficients of ϕ or α_x^2, being careful to drop in this substitution the numerical factor of each coefficient, the result vanishes identically, whatever the value of y_1, y_2. In other words, the second transvectant of $\overline{F}$ over ϕ vanishes identically,

$$(\overline{A}\alpha)^2 \overline{A}_y^{n-2} \equiv 0.$$

Introducing technical terms first used by Rosanes* and Reye,† I may say:

(2) the quantic $\overline{F}$ is *conjugate* or *apolar* to the quadric ϕ.

My present interest in the covariant $\overline{F}$ arises from the fact that, as $\overline{F}$ was derived from R by evection, so, inversely, the resultant itself can be obtained from $\overline{F}$ immediately by trans-

* *Journal für Math.*, vol. 75 (1873).
† " " " vol. 72 (1870).

vection over f n times. Accordingly I shall consider in their order the following questions:

(1) What symbolic products can enter in the expression of the resultant R by fundamental invariants?

(2) Consequently, what symbolic products can enter in the expression of the covariant $\overline{F}$ by fundamental covariants?

(3) In order that the quantic $\overline{F}$ may be *apolar* to the quadric ϕ, what must be the numerical multipliers of these symbolic products?—This amounts to the determination of arbitrary constants by means of a differential equation.

(4) What, finally, is the consequent expression for the resultant R?

For the sake of simplicity I will consider f as a quantic of *even* order: $f = a_x^n = a_x^{2p} = b_x^{2p}$.

The first question is answered by applying two elementary theorems: first, that if a symbolic factor (ab) or $(\alpha\beta)$ containing two symbols of the same quantic occur in any term, it must occur to the second or some other even power; second, that a factor $(a\alpha)$ containing symbols of different quantics can occur only to an even power, i.e. to the second power, for the only alternative is the appearance of the four factors $[(a\alpha)(b\alpha)(a\beta)(b\beta)]$, and this product is identically equal to the following:

$$\tfrac{1}{2}\{(a\alpha)^2(b\beta)^2 + (a\beta)^2(b\alpha)^2 - (ab)^2(\alpha\beta)^2\}.$$

Therefore R is composed of not more than these $\frac{n}{2}+1$ terms:

$$R = \left\{\begin{array}{l} l_0 \,.\, (ab)^{2p} \,.\, [(\alpha\beta)^2]^p \\ + l_1 \,.\, (ab)^{2p-2} \,.\, (a\alpha)^2(b\beta)^2 \,.\, [(\delta\gamma)^2]^{p-1} \\ + l_2 \,.\, (ab)^{2p-4} \,.\, (a\alpha)^2(a\beta)^2(b\gamma)^2(b\delta)^2 \,.\, [(\epsilon\zeta)^2]^{p-2} \\ + \text{etc.} \\ + l_p \,.\, (a\alpha)^2(a\beta)^2 \ldots (\text{to } p \text{ factors}) \,.\, (b\gamma)^2 \,.\, (b\delta)^2 \ldots (\text{to } p \text{ factors}). \end{array}\right\}$$

The second question is answered, since this formula for R is symmetrical in symbols a and b, by replacing the b's with x's as heretofore. We find in this way:

$$\tfrac{1}{2} \,.\, \overline{F} = \left\{\begin{array}{l} l_0 \,.\, a_x^{2p} \,.\, [(\alpha\beta)^2]^p \\ + l_1 \,.\, a_x^{2p-2}(a\alpha)^2 \,.\, \beta_x^2 \,.\, [(\gamma\delta)^2]^{p-1} \\ l_2 + \,.\, a_x^{2p-4}(a\alpha)^2(a\beta)^2 \,.\, \gamma_x^2\delta_x^2[(\epsilon\zeta)^2]^{p-2} \\ + \text{etc.} \\ + l_p \left\{\begin{array}{l} (a\alpha)^2(a\beta)^2 \ldots (\text{to } p \text{ factors}). \\ \gamma_x^2 \,.\, \delta_x^2 \ldots (\text{to } p \text{ factors.} \end{array}\right\} \end{array}\right\}$$

This formula contains, I may remark in passing, a theorem which ought to find a direct proof through skilful manipulation of the determinant expression for $\overline{F}$. It is this:

When the form f is of even order, the evectant $\overline{F}$ of its resultant with a quadric ϕ is expressible in the form

$$\overline{F} \equiv A \,.\, f + B \,.\, \phi,$$

where A is a rational integral function of coefficients of the quadric ϕ and independent of the variables, while B is of the order $(n-2)$ in the variables, and likewise rational and integral in the coefficients of f and of ϕ.

Taking up the third question, we have only to apply combinatory enumeration and the identity

$$(\alpha\beta)(a\beta)a_x = \tfrac{1}{2}(\alpha\beta)^2 a_x$$

in order to form directly from the development of $\overline{F}$ on p. 13 the second transvectant $(\overline{A}\alpha)^2 \overline{A}_y^{n-2}$, whose identical vanishing yields the equations of condition:

$$l_0 + \frac{2}{n} l_1 = 0,$$

$$\frac{(n-2)(n-3)}{2} l_1 + 2(n-2) l_2 = 0,$$

$$\frac{(n-4)(n-5)}{2} l_2 + 3(n-3) l_3 = 0,$$

etc., etc.

From these, calling for convenience $l_0 = 1$, we find for the numerical constants these values:

$$l_1 = -\frac{n}{2} = p,$$

$$l_2 = +\frac{n(n-3)}{2^2 \,.\, 2 \,.\, 1},$$

$$l_3 = -\frac{n(n-4)(n-5)}{2^3 \,.\, 3 \,.\, 2 \,.\, 1},$$

$$\vdots$$

$$l_p = (-1)^p \,.\, \frac{2p \,.\, \underline{|p-1}}{2^p \,.\, \underline{|p}} = \frac{(-1)^p}{2^{p-1}}.$$

Thus we have the constants needed in writing as a covariant of a_x^{2p} and a_x^2 the quantic

$$\overline{A}_x^{2p} = a_x^{2p} + \lambda_x^{2p-2} \cdot a_x^2,$$

which is required to be apolar (or conjugate) to the quadric a_x^2.

Fourthly, this gives for the resultant R exactly the expression found by Clebsch (*Binäre Formen*, p. 88). Writing, in our formula on p. 13, Δ for the discriminant $(\alpha\beta)^2$ of the quadric, and inserting the above-found numerical values of the constants $l_0, l_1, l_2, \ldots l_p$, I have the final formula (neglecting possibly a multiplicative constant)

$$\begin{aligned} R = {} & \Delta^p (ab)^{2p} \\ & - \frac{2p}{2} \cdot \Delta^{p-1} \cdot (ab)^{2(p-1)} (a\alpha)^2 (b\beta)^2 \\ & + \frac{2p(2p-3)}{2^2 \cdot 2!} \Delta^{p-2} \cdot (ab)^{2(p-2)} \cdot (a\alpha)^2 (a\beta)^2 (b\gamma)^2 (b\delta)^2 \\ & + \text{etc.} + \text{etc.} \\ & + \frac{(-1)^p}{2^{p-1} \cdot \lfloor p} \left\{ \begin{array}{l} (a\alpha)^2 (a\beta)^2 \ldots \text{(to } p \text{ factors).} \\ (b\gamma)^2 (b\delta)^2 \ldots \text{(to } p \text{ factors).} \end{array} \right\}. \end{aligned}$$

Had we assumed the order of f as odd, the process would require but one principal change,—the substitution of the Jacobian of f and ϕ,

$$(a\alpha) a_x^{n-1} \alpha_x = g,$$

instead of f throughout the formulæ for R and $\overline{F}$, and so of course in the theorem mentioned (p. 13) as deserving of a more direct proof. This change affects the constants of the formula, which again come to agree with those given by Clebsch.

In closing I may add that the coincidence of the semicombinant property and the apolar property, which appeared quite incidentally in our discussion of the covariant $\overline{F}$ (p. 12), is demonstrably a necessary coincidence quite apart from the special form of the covariant. On this necessary union of the two properties in covariants of a certain order, indeed, is based the entire theory of semicombinant invariants and covariants of a system of quantics.

NORTHWESTERN UNIVERSITY, *August* 4, 1894.

THE MEETING OF THE AMERICAN ASSOCIATION.

THE forty-third annual meeting of the American Association for the Advancement of Science was held at Brooklyn, N. Y., August 15–24, Daniel G. Brinton, of Media, Pa., presiding. The attendance was large, the total registration being 477. The number of new members admitted was 213.

The members enjoyed excursions to Long Branch, to points of interest in New York Harbor, to Cold Spring Harbor, and to West Point on the Hudson.

Astronomy was much better represented by the papers presented to Section A than was pure mathematics. The officers of the section were: Edgar Frisby, Washington, D. C., Vice-President; Jefferson E. Kershner, Lancaster City, Pa., Secretary; R. S. Woodward, New York, N. Y., Councilor; C. A. Doolittle, South Bethlehem, Pa., C. A. Waldo, Greencastle, Ind., C. H. Rockwell, Tarrytown, N. Y., Ormond Stone, Charlottesville, Va., J. A. Brashear, Alleghany, Pa., together with the Vice-President and Secretary, Sectional Committee; George B. Halsted, Austin, Texas, Member of Nominating Committee; L. G. Weld, Iowa City, Iowa, J. Burkitt Webb, Hoboken, N. J., George D. Olds, Amherst, Mass., together with the Vice-President and Secretary, Sectional Nominating Committee; George B. Halsted, Austin, Texas, Press Secretary.

The following papers were presented:

1. Some attempts to photograph the solar corona without an eclipse. By George B. Hale, Chicago, Ill.

2. On the control of the equatorial clock. By G. W. Hough, Evanston, Ill.

3. Requisites for governing the motion of equatorial telescopes. By W. R. Warner, Cleveland, O.

4. Recent latitude determination at the Sayre Observatory. By C. A. Doolittle, South Bethlehem, Pa.

5. A simple proof of Radares symmetrical form of the differential equations of the relative motion of the planets. By Ormond Stone, Charlottesville, Va.

6. A configuration of 36 points, 27 lines, 36 planes, a special case of which leads to Klein's hyperelliptic configuration of 40 points, 90 lines, 40 planes. By E. Hastings Moore, Chicago, Ill.

7. Stellar photometry. By Henry M. Parkhurst, Brooklyn, N. Y.

8. An extension of the Gaussian-potential theory of terrestrial magnetism. By H. A. Bauer. Presented by R. S. Woodward, New York, N. Y.

9. Sketch of zone observations at the Naval Observatory. By Aaron N. Skinner, Washington, D. C.

10. Gilliss's catalogue of Southern stars. By Edgar Frisby, Washington, D. C.

No. 1 opened with a review of the experiments of Langley, Huggins, and others, made in the hope of observing the corona without an eclipse. The author's experiments were begun at the Kenwood Observatory, Chicago, in 1892, and have since been continued on Pike's Peak and Mount Etna. The instrument employed was the spectroheliograph, which allows the photographic plate to be protected from the direct light of the sky by shading it with the dark band K of the solar spectrum. The light of the corona being represented in large part by a continuous spectrum, the method employed permits the brightness of the sky to be cut down without appreciably impairing the brightness of the corona. The experiments on Pike's Peak and Mount Etna were rendered useless by smoke, in the first case from forest fires, and in the second from the volcano. Two new methods of observing the corona without an eclipse were also described.

No. 2 was a description of a method of controlling an equatorial clock as applied by the author to the instrument in the observatory at Evanston, Ill. It is one of many improvements devised by the author, the adaptation of which at Evanston has added greatly to the equipment of the observatory.

No. 3 stated that the prime requisites in the driving-clock are: first, that the pendulum shall be perfectly free to maintain its exact theoretical position, whether the resistance to its motion be much or little; second, that whatever its theoretical position may be it should be mounted isochronously, so that its time of revolution will be the same, whether much or little of the driving power is used to move the telescope itself. The mechanism to absorb the excess of power must be so arranged that the power not required elsewhere will be instantly transferred to it and absorbed, thus leaving the pendulum revolving at its normal rate.

Two methods for accomplishing these desiderata were given. One by Professor Young of Princeton in which the pendulum-rod is hung, so that its varying angle leaves its theoretical length practically constant, and another as applied by Repsold to the heliometer at Yale, by the use of the principle of the tuning-fork.

It was pointed out that the governing of the slow motion in right ascension has been well accomplished by use of the method usually applied to slow motion in declination.

No. 4 was a comparison of the results of observations of the variation of latitude at the Sayre Observatory with those deduced from the formula of Dr. S. C. Chandler. This formula for the Sayre Observatory is

$$\phi - \phi_0 = -0.156'' \cos(t - T_1') \, 0.850° - 0.153'' \cos(\odot - 280°),$$

where ϕ = value of the latitude at any time t;
ϕ_0 = mean value of the latitude;
$\odot$ = the sun's longitude;
T_1' = 2,412,125 (Julian date).

The term $-0.156''\cos(t - T_1')0.850°$ is that part of the variation having a period of 424 days; the variation represented by $-0.153''\cos(\odot - 280°)$ has a period of one year. It was shown that the complete formula did not compare very favorably with the results of observation, but that when only the 424-day term was considered the agreement was very good. This is in accord with Chandler's latest investigations, which show that the annual movement is in a very excentric ellipse. The German observatories, being nearly on the direction of the major axis of this ellipse, experience nearly the full effect of the movement, while for the meridian of Bethlehem it nearly vanishes. The paper was accompanied by a chart.

No. 5 was a shortening and simplification of what has been heretofore a long and laborious proof. The abridgment of demonstrations is an important labor, as it seems to offer the only solution of the problem of acquiring, with the rapidly-extending domain of mathematical science, a fair proportion of its content in a lifetime.

No. 6 was the discussion of a configuration which is determined by an arbitrary fundamental tetrahedron and an arbitrary transversal (not lying in a face or passing through a vertex of the tetrahedron). These elements determine a Reye's tetrahedral complex whose lines cut the faces [are joined to the vertices] of the tetrahedron in ranges of four points [axial pencils of four planes] having the same anharmonic ratio. The 27 Cf.-lines belong to the complex, the transversal being a Cf.-line. The 36 Cf.-points lie [Cf.-planes pass] by fours on [through] the 27 Cf.-lines and by nines on [through] the four faces [vertices] of the fundamental tetrahedron.

If in particular the transversal cuts the four faces in a range of four points having an *equianharmonic* ratio, then by adjoining certain elements to our configuration we obtain Klein's hyperelliptic configuration* of 40 points, 90 lines, 40 planes.

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* WITTING: Über eine der Hesse'schen Configuration der ebenen Curve dritter Ordnung analoge Configuration im Raume auf welche die Transformationstheorie der hyperelliptischen Functionen führt. Inaugural-dissertation, Dresden, 1887.

MASCHKE: Aufstellung des vollen Formensystems einer quaternären Gruppe von 51840 linearen Substitutionen: *Mathematische Annalen*, vol. 33, pp. 317–344, 1888.

No. 7 was a reply to the criticisms in the *Astronomische Nachrichten* of the results obtained by the Harvard Observatory with the meridian photometer. The necessity was shown of photometric methods as a basis for the division of the stars into magnitudes, especially for the reason that errors of judgment in the estimation of brightness will inevitably be perpetuated, while errors in photometric observation will be gradually eliminated. From the author's independent photometric observations with a different instrument, he had verified the general accuracy of the observations with the meridian photometer, and attributed the errors which had been detected chiefly to the misidentification of stars, a source of error to which he considered the meridian photometer peculiarly liable in the observation of variable stars. This confined the main objection to less than a dozen pages contained in a supplement to one of two large volumes of meridian photometer observations. A defence was made of what had been styled "excessive haste" in the Harvard observations, upon the ground that at the time when they were made, when there were no determinations for faint stars, it was important to obtain observations of as many stars as possible, in order that they might be easily accessible, even at the risk of errors and inaccuracies which could be easily sifted out. This work stands in the same relation to later and more accurate observations that the "Durchmusterung" bears to meridian-circle observations with regard to stellar positions.

No. 8 showed how to utilize observations with a view to determining the time-variation of the earth's magnetic potential. The idea of the paper is to derive values for the potential at different epochs, separated by intervals of a century or more, and thus discover its law of change. A difficulty met in the application of the idea arises from the paucity of data in the earlier observations, these consisting generally of a single one of the elements, namely, declination, dip, or horizontal force. The paper shows, nevertheless, how to use such incomplete data, and the author hopes by an application of his method to throw additional light on the obscure question of the secular variation of the elements of the earth's magnetism.

No. 9 opened with an account of Argelander's "Northern Durchmusterung" and the organization by the German Astronomical Society of the work of accurately determining the positions of all stars there given to the 9th magnitude inclusive. The German Astronomical Society in undertaking the same task for the stars in Schönfeld's "Southern Durchmusterung" assigned to the Naval Observatory at Washington the zone $-13^\circ\ 50'$ to $-18^\circ\ 10'$ containing 8689 of these stars.

Since January 13, 1894, when the work commenced, 5105

zone and about 600 zero and azimuth stars have been observed.

For a working list the card system has been used with great satisfaction. A card is devoted to each star, with a space for the results of observation. In arranging the cards for an evening's work, slips are inserted to indicate the time for reading meteorological instruments and for lunch.

Fifty to sixty stars are observed per hour. The programme requires two accordant observations of each zone star. The observations are entirely differential and are founded upon a list of 303 zero stars.

No. 10 was an account of the work of reduction of the data of Gilliss upon the positions of stars in the southern hemisphere. The author described the difficulties encountered by the observer from poor equipment and insufficient support.

Among the errors of observation were the recording of one cross wire for another, and use of the telescope when its cross wires were oblique, the error thus arising being attributed to refraction.

Mr. J. A. Brashear of Allegheny, Pa., exhibited a concave grating with 15,000 lines to the inch, ruled on Professor Rowland's new ruling-machine.

The place of the next annual meeting will probably be San Francisco, although no final decision was reached, followed by a meeting at Buffalo in 1896. The President of the next meeting will be E. W. Morley, Cleveland, Ohio.

The officers of section A chosen for 1895 are: E. S. Holden, Mt. Hamilton, Cal., Vice-President, and E. H. Moore, Chicago, Ill., Secretary.

COLUMBIA COLLEGE. E. M. BLAKE.

NOTES.

THE meeting of the British Association for the Advancement of Science was held this year at Oxford, beginning on August 8 with the presidential address by Lord Salisbury. The attendance was exceptionally large, many eminent foreign scientists being present, and a number of papers of high importance was read. The University expressed its appreciation of the presence of so many distinguished foreigners by conferring the honorary degree of D.C.L. upon some twelve of them, among whom was Professor Gosta Mittag-Leffler. On Thursday, August 9, a soirée and exhibition of scientific apparatus was held in the University Museum. Among the exhibits was a series of linkage models by Professor Henrici. On Friday the section of mathematics and physics held a joint session with the section of mechanics, and several discussions

were engaged in, one of the most interesting being on "Integrators, harmonic analysers and integraphs, and their application to physical and engineering problems."

THE courses in pure mathematics announced at the University of Berlin for the semester beginning October 15 include the following: Fuchs, Linear differential equations, Elliptic functions—Frobenius, Theory of algebraic equations—Schwarz, Synthetic conic sections, Integral Calculus, Calculus of variations—Hensel, Theory of numbers, Theory of algebraic integrals—Hettner, Fourier's series—Knoblauch, Curved surfaces, Twisted curves, Differential calculus—Schlesinger, Theory of definite integrals. T. S. F.

THE first and second sections of the French Association for the Advancement of Science at the Caen Congress, in their session of August 14, 1894, when the order of the day was, a study of means to insure a more facile and fruitful exchange of views between mathematicians of different nations, and thus to contribute to the progress of mathematics and the perfecting of its methods, passed unanimously the following resolutions, after a thorough discussion, joined in by many members:

1. We give our most complete adherence to the plan for creating *international mathematical congresses*, and declare ourselves disposed to bear all aid to the efforts making or to be made toward this end.

2. We approve absolutely the idea of Mr. Mansion, relative to the making of *mathematical vocabularies*, and applaud the beginning of its realization already accomplished by Commandant Brocard in the preparation of a French mathematical vocabulary.

3. We express the hope that the project of Mr. Jacques Boyer concerning the establishment of a *mathematical dictionary* may come to a happy issue, both in France and in many other countries.

4. We believe we should draw attention to the remarkable mathematical monographs being published at this moment in Germany, and of which it would be very desirable to see published translations into divers languages.

5. We consider that the great efforts made by Professor Peano and many of his confrères for the propagation of *algorithmic logic* and the publication of a *mathematical formulary* are of a nature to contribute powerfully to the end we seek to attain.

6. We are happy to state the degree of advancement of the *Répertoire Bibliographique des Sciences Mathématiques*, and, in the same order of ideas, to applaud the highly interesting publication due to a group of mathematicians in Holland,

particularly Mr. P. H. Schoute, which is entitled *Revue Semestrielle des Publications Mathématiques.*

7. We think that the publication of *l'Intermédiaire des Mathématiciens*, since the beginning of 1894, has rendered and will render very great services in what concerns the relations of mathematicians to each other; we express our gratitude to the founders, Laisant and Lemoine, and felicitate ourselves that this initiative was due to two members of the French Association for the Advancement of Science.

8. We take into very serious consideration the reflections presented by Mr. Lémeray on the possibility of establishing mathematical libraries, having for object to put books at the disposal of workers remote from scientific centres.

9. We decide that the question, under the general form which has been given it, shall be retained as the order of the day for sessions at the Bordeaux meeting in 1895. G. B. H.

THE Prince Jablonowski Society of Leipzig makes the following announcements in regard to prizes in the department of mathematics and natural science:

For 1894.—The determination afforded by Leverrier of the secular perturbations of orbits, in the case of the inner planets particularly, gives unsatisfactory results, inasmuch as the terms of the second order cannot be used in the calculations, they being inexact and occasionally even greater than the terms of the first order. This difficulty, which in its consequences seems to be related to certain anomalies in the motion of Mercury relative to its perihelion, Leverrier* was inclined to attribute to the method of treatment adopted, by which in the first approximation the differential equations of the problem were considered as linear. The society desires accordingly *a new determination of the secular perturbations of at least the orbits of Mercury, Venus, the Earth, and Mars, having regard to the terms of higher order*, the methods employed to be free from objections, and the difficulties encountered by Leverrier, which would destroy the utility of the results, to be removed.

For 1897.—The methods which Monge, Ampère, and Darboux have furnished for integrating partial differential equations of the second and higher orders are applicable only to those equations which have in common with other equations solutions dependent not merely upon arbitrary constants. On the other hand it follows from Lie's investigations upon infinite groups that equations which admit an infinite group of contact transformations, in general, bear to other equations the relation just described (involutorial rela-

* Recherches astronomiques, chap. IX, art. 16, and Additions III, p. 51.

tion).* The society desires *that the methods of integration resulting from this remark should be developed, and should be illustrated to as great an extent as possible by instructive examples fully worked out.*

The prizes are of 1000 marks each. Papers offered in competition will be received by the secretary of the society until November 30 of the year indicated. The results will be announced in the following March or April.

PRINCE BALDASSARE BONCOMPAGNI, for many years editor of the *Bulletino di Bibliografia e di Storia delle Scienze matematiche e fisiche*, died at an advanced age in Rome on April 12 last.

HERMANN LUDWIG FERDINAND BARON VON HELMHOLTZ died in Berlin on September 8, at the age of seventy-three years.

T. S. F.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

ANTILOTTI (F.). Trattato di analisi algebrica. Parte I: Analisi algebrica elementare. Napoli, 1893. 8vo. 9 and 308 pp.

ARCAIS (F. D'). Corso di calcolo infinitesimale. Vol. II: Calcolo integrale. Padova, 1894. 8vo. Fr. 11.00

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